4.4 Simplifying Algebraic Expressions Involving Radicals

When a radical contains a variable, we must consider RESTRICTIONS on the variable.

(Are there any values that ***x*** is not allowed to be?)

NOTE:

1. You cannot take the square root of a negative number!

2. You cannot divide by zero.

Let's take a look at the following:

$1.a)\ \sqrt{x}$ $b)\ \sqrt{x-3}$ $c)\ \sqrt{x+2}$

What is the difference between (abc) and (def)?

How does that change its restrictions?

$d)\ \frac{1}{\sqrt{x}}$ $e)\ \frac{1}{\sqrt{x-3}}$ $f)\ \frac{1}{\sqrt{x+2}}$

1. a) $\sqrt{x}$ b) $\sqrt{x-3}$ c) $\sqrt{x+2}$

a) $x \geq 0$

b) $x-3 \geq 0$, $x \geq 3$

c) $x+2 \geq 0$, $x \geq -2$

What is the difference between (abc) and (def)?

How does that change its restrictions?

You cannot divide by 0!

d) $\frac{1}{\sqrt{x}}$ e) $\frac{1}{\sqrt{x-3}}$ f) $\frac{1}{\sqrt{x+2}}$

d) $x > 0$

e) $x-3 > 0$, $x > 3$

f) $x+2 > 0$, $x > -2$

Since we already know that √x always refers to the principal or positive square root, we will note the following:

$$\sqrt{x^2} = |x|$$

$|x|$ is called the Absolute Value of ***x*** and refers to its distance from zero, which is ***always positive***.

Example: $\sqrt{(-4)^2} = |-4| = 4$ because $\sqrt{(-4)^2} = \sqrt{16} = 4$

Using the Laws of Exponents, we recall the following facts:

$$x^2 \cdot x^2 = x^4$$
$$x^3 \cdot x^3 = x^6$$
$$x^4 \cdot x^4 = x^8$$
$$x^5 \cdot x^5 = x^{10}$$

Exponent Law:

When ***multiplying*** two powers that have the same base, we **ADD** the exponents.

This helps to explain how each square root below is simplified:

$$\sqrt{x^4} = \sqrt{x^2 \cdot x^2} = x^2$$
$$\sqrt{x^6} = \sqrt{x^3 \cdot x^3} = x^3$$
$$\sqrt{x^8} = \sqrt{x^4 \cdot x^4} = x^4$$
$$\sqrt{x^{10}} = \sqrt{x^5 \cdot x^5} = x^5$$

OBSERVE:
The exponent on each square root is one half the exponent on the radicand.

What happens when the exponent on the radicand is not even?

$$\sqrt{x^7} = \sqrt{x^6 \cdot x} = \sqrt{x^6} \cdot \sqrt{x} = x^3\sqrt{x}$$

We must factor the radicand into an ***x*** with an even exponent multiplied by a single ***x***.

Examples: (What are the restrictions on each variable?)

2.a) $\sqrt{x^{13}}$

$= \sqrt{x^{12} \cdot x}$

$= x^6\sqrt{x}$

b) $\sqrt{y^{25}}$

$= \sqrt{y^{24} \cdot y}$

$= y^{12}\sqrt{y}$

Now we can extend this to examples that include mixed radicals:

3.a) $\sqrt{9x^{11}}$

b) $\sqrt{12x^7y^{16}}$

c) $2\sqrt{28x^{13}}$

d) $3\sqrt{6x^8y^2}$

What are the restrictions on each variable above?

Now we can extend this to examples that include mixed radicals:

$3.a)\ \sqrt{9x^{11}}$

$=\sqrt{9x^{10}\cdot x}$

$=3x^5\sqrt{x}$

$b)\ \sqrt{12x^7y^{16}}$

$=\sqrt{4\cdot 3\cdot x^6\cdot x\cdot y^{16}}$

$=2x^3y^8\sqrt{3x}$

$c)\ 2\sqrt{28x^{13}}$

$=2\sqrt{4\cdot 7\cdot x^{12}\cdot x}$

$=4x^6\sqrt{7x}$

$d)\ 3\sqrt{6x^8y^2}$

$=3x^4y\sqrt{6}$

What are the restrictions on each variable above?

Operations With Radicals Containing Variable Expressions

We use the same principles that we used for numerical radicands.

Adding/Subtracting:

NOTE: Simplify first, then combine like radicals.

$4.a)\ 3\sqrt{x}+5\sqrt{x}-\sqrt{x}$

$b)\ \sqrt{8x^5}+3\sqrt{50x^5}$

$c)\ 5\sqrt{x^6}+2\sqrt{y^4}-\sqrt{x^6}+4\sqrt{y^4}$

What are the restrictions on each variable above?

4.a) $3\sqrt{x} + 5\sqrt{x} - \sqrt{x}$

$= 7\sqrt{x}$

b) $\sqrt{8x^5} + 3\sqrt{50x^5}$

$= \sqrt{4 \cdot 2 \cdot x^4 \cdot x} + 3\sqrt{25 \cdot 2 \cdot x^4 \cdot x}$

$= 2x^2\sqrt{2x} + 15x^2\sqrt{2x}$

$= 17x^2\sqrt{2x}$

c) $5\sqrt{x^6} + 2\sqrt{y^4} - \sqrt{x^6} + 4\sqrt{y^4}$

What are the restrictions on each variable above?

Multiplying:

NOTE: Multiply outsides, then insides.

5.a) $\left(5\sqrt{x}\right)\left(-4\sqrt{x^3}\right)$

b) $\left(2\sqrt{x}+1\right)\left(3-6\sqrt{x}\right)$

What are the restrictions on each variable above?

$5.a)\ \left(5\sqrt{x}\right)\left(-4\sqrt{x^3}\right)$

$=-20\sqrt{x^4}$

$=20x^2$

$b)\ \left(2\sqrt{x}+1\right)\left(3-6\sqrt{x}\right)$

$=6\sqrt{x}-12\sqrt{x^2}+3-6\sqrt{x}$

$=-12x+3$

What are the restrictions on each variable above? $x\geq 0$

Dividing:

NOTE: Combine into a single radical if possible, and then simplify, or rationalize the denominator as necessary.

$6.a)\ \frac{\sqrt{10x^9}}{\sqrt{5x^3}}$

$b)\ \frac{\sqrt{2}}{\sqrt{5x}}$

6.a) $\dfrac{\sqrt{10x^9}}{\sqrt{5x^3}}$

$=\sqrt{\dfrac{10x^9}{5x^3}}$

$=\sqrt{2x^6}$

$=x^3\sqrt{2}$

b) $\dfrac{\sqrt{2}}{\sqrt{5x}}$

$=\dfrac{\sqrt{2}}{\sqrt{5x}}\cdot\dfrac{\sqrt{5x}}{\sqrt{5x}}$

$=\dfrac{\sqrt{10x}}{5x}$

c) $\dfrac{10}{3\sqrt{x^3}}$

d) $\dfrac{4+2\sqrt{x}}{\sqrt{x}}$

$c)\ \dfrac{10}{3\sqrt{x^3}}$

$= \dfrac{10}{3\sqrt{x^3}} \cdot \dfrac{\sqrt{x^3}}{\sqrt{x^3}}$

$= \dfrac{10\sqrt{x^3}}{3x^3}$

$d)\ \dfrac{4+2\sqrt{x}}{\sqrt{x}}$

$= \dfrac{4+2\sqrt{x}}{\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}}$

$= \dfrac{4\sqrt{x}+2x}{x}$

Assign:

p. 211 - 213, # 1-6, 8-12