4.3 Multiplying and Dividing Radicals

NOTE:

You can only multiply or divide radicals

IF they have the same index.

Property #1:

The product of two square roots is equal to the square root of the product.

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

1.a) $\sqrt{5} \cdot \sqrt{2} = \sqrt{5 \cdot 2} = \sqrt{10}$

b) $\sqrt{6} \cdot \sqrt{3} = \sqrt{6 \cdot 3} = \sqrt{18}$

NOTE: Be Careful!

$\sqrt{5} \cdot 2 \neq \sqrt{10}$

$\sqrt{5} \cdot 2 = 2\sqrt{5}$

Property #2:

The product of two mixed radicals is equal to the product of the rational numbers times the product of the radicals.

$$c\sqrt{a}\cdot d\sqrt{b} = cd\sqrt{ab}$$

$2.a)\ 3\sqrt{2}\cdot 4\sqrt{5}$

$= 3\cdot 4\sqrt{2\cdot 5}$

$= 12\sqrt{10}$

$b)\ 7\sqrt{6}\cdot 2\sqrt{3}$

$= 7\cdot 2\sqrt{6\cdot 3}$

$= 14\sqrt{18}$

$= 14\sqrt{9\cdot 3}$

$= 14\cdot 3\sqrt{3}$

$= 42\sqrt{3}$

NOTE:

Always write in simplest form!

$c)\ \sqrt{12}\cdot\sqrt{18}$

NOTE: Can multiply then simply

OR can simplify then multiply!

Could/should we simplify first, and then multiply??

$c)\ \sqrt{12} \cdot \sqrt{18}$

NOTE: Can multiply then simply

OR can simplify then multiply!

Method 1:

$\sqrt{12} \cdot \sqrt{18}$
$= \sqrt{12 \cdot 18}$
$= \sqrt{216}$
$= \sqrt{36 \cdot 6}$
$= 6\sqrt{6}$

OR

Method 2:

$\sqrt{12} \cdot \sqrt{18}$
$= 2\sqrt{3} \cdot 3\sqrt{2}$
$= 6\sqrt{6}$

It is recommended to simplify first to avoid mistakes.

Could/should we simplify first, and then multiply??

$d)\ \sqrt{32} \cdot \sqrt{20}$

$e)\ 2\sqrt{27} \cdot 5\sqrt{45}$

$d)\ \sqrt{32}\cdot\sqrt{20}$

$=4\sqrt{2}\cdot 2\sqrt{5}$
$=8\sqrt{10}$

$e)\ 2\sqrt{27}\cdot 5\sqrt{45}$

$=2\cdot 3\sqrt{3}\cdot 5\cdot 3\sqrt{5}$
$=6\sqrt{3}\cdot 15\sqrt{5}$
$=90\sqrt{15}$

$f)\ 7\sqrt{2}\cdot 3\sqrt{2}$

$f)\ 7\sqrt{2} \cdot 3\sqrt{2}$

$= 21\sqrt{4}$

$= 21 \cdot 2$

$= 42$

NOTE:

When the radicand is the same number, the radical becomes a rational number!

Property #3:

You can use the same properties you use with rational numbers to multiply and divide radical numbers.

ie, distributive property and FOIL

$3.a)\ 4\sqrt{2}\left(7\sqrt{5} + \sqrt{3}\right)$

$= 28\sqrt{10} + 4\sqrt{6}$

Note the similiarity!

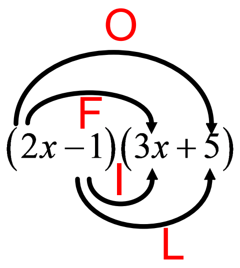
$4x(7x + 3)$

$= 28x^2 + 12x$

$b)\ 4\sqrt{10}\left(3\sqrt{7}-6\right)$

$=12\sqrt{70}-24\sqrt{10}$

Remember FOIL?



$= 6x^2 + 10x - 3x - 5$

$= 6x^2 + 7x - 5$

$$c)\left(3+\sqrt{14}\right)\left(4\sqrt{7}-5\sqrt{2}\right)$$

$$=12\sqrt{7}-15\sqrt{2}+4\sqrt{98}-5\sqrt{28}$$

$$=12\sqrt{7}-15\sqrt{2}+4\cdot 7\sqrt{2}-5\cdot 2\sqrt{7}$$

$$=12\sqrt{7}-15\sqrt{2}+28\sqrt{2}-10\sqrt{7}$$

$$=2\sqrt{7}+13\sqrt{2}$$

$$d)\ \left(\sqrt{2}-10\sqrt{3}\right)^2$$

$$=\left(\sqrt{2}-10\sqrt{3}\right)\cdot\left(\sqrt{2}-10\sqrt{3}\right)$$

$$=\sqrt{4}-10\sqrt{6}-10\sqrt{6}+100\sqrt{9}$$

$$=2-20\sqrt{6}+100(3)$$

$$=302-20\sqrt{6}$$

Assign:

p. 198, #1, 4, 5, 6a, 8, 11

Property #4:

The quotient of two square roots is equal to the square root of the quotient.

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$4.a)\sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2}$

$b)\ \frac{\sqrt{56}}{\sqrt{7}} = \sqrt{\frac{56}{7}} = \sqrt{8} = 2\sqrt{2}$

Property #5:

When two radicals are divided and the numbers under the radical signs cannot be divided evenly, we must use a process called...

Rationalizing the Denominator

...which converts the denominator to a rational number.

*** Multiply the numerator and the denominator by the radical in the denominator.*

Should we simplify first??

$5.a)\ \frac{\sqrt{2}}{\sqrt{5}}$

$= \frac{\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{10}}{\sqrt{25}}$

$= \frac{\sqrt{10}}{5}$

$b)\ \frac{\sqrt{7}}{\sqrt{3}}$

$= \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{21}}{\sqrt{9}}$

$= \frac{\sqrt{21}}{3}$

Property #6:

When two mixed radicals are divided, we can divide the "outsides" and divide the radicals.

$$\frac{c\sqrt{a}}{d\sqrt{b}} = \frac{c}{d}\sqrt{\frac{a}{b}}$$

6.a) $\frac{24\sqrt{14}}{3\sqrt{2}}$

$= 8\sqrt{7}$

b) $\frac{6\sqrt{30}}{-12\sqrt{3}}$

$= -\frac{1}{2}\sqrt{10} = \frac{-\sqrt{10}}{2}$

Mixed radicals may also require Rationalizing of the Denominator if the radicals do not divide evenly.

c) $\frac{7\sqrt{10}}{5\sqrt{3}}$

$= \frac{7\sqrt{10}}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{7\sqrt{30}}{5\sqrt{9}}$

$= \frac{7\sqrt{30}}{15}$

Multiply by $\sqrt{3}$ not $5\sqrt{3}$

d) $\frac{14\sqrt{8}}{10\sqrt{3}}$

$= \frac{14 \cdot 2\sqrt{2}}{10\sqrt{3}}$

$= \frac{14\sqrt{2}}{5\sqrt{3}}$

$= \frac{14\sqrt{2}}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{14\sqrt{6}}{15}$

TIP:

Reduce radicals first before rationalizing!

Radicals may be added/subtracted in the numerator of a fraction that also requires Rationalizing of the Denominator.

$e)\ \frac{2\sqrt{3}+4\sqrt{5}}{\sqrt{6}}$

$f)\ \frac{-5\sqrt{8}+\sqrt{11}}{7\sqrt{3}}$

$e)\ \frac{2\sqrt{3}+4\sqrt{5}}{\sqrt{6}}$

$$=\frac{2\sqrt{3}+4\sqrt{5}}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{2\sqrt{18}+4\sqrt{30}}{6}$$

$$=\frac{6\sqrt{2}+4\sqrt{30}}{6}$$

$f)\ \frac{-5\sqrt{8}+\sqrt{11}}{7\sqrt{3}}$

$$=\frac{-10\sqrt{2}+\sqrt{11}}{7\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{-10\sqrt{6}+\sqrt{33}}{21}$$

Assign:

p. 198 - 199, #2, 13 - 16, 19

+ Tarsia Puzzle

Mid-Chapter Review

Quiz